



MicroWELT
**Microsimulation of Disaggregated National Transfer Accounts (NTAs) for the
Comparative Study of Welfare State Regimes**

Martin Spielauer

Austrian Institute of Economic Research
martin.spielauer@wifo.ac.at

Meritxell Solé,

University of Barcelona
meritxell.sole@ub.edu

Thomas Horvath,

Austrian Institute of Economic Research
thomas.horvath@wifo.ac.at

Marian Fink

Austrian Institute of Economic Research
marian.fink@wifo.ac.at

Concepció Patxot

University of Barcelona
cio.patxot@ub.edu

Guadalupe Souto

University of Barcelona
guadalupe.souto@uab.es

ABSTRACT: This research note introduces the MicroWELT model developed alongside an international research program studying the distributional effects along the lifecycle of four welfare state regimes in the context of demographic change. MicroWELT is a portable continuous time interacting population model based on data readily available for many countries, most importantly the Euromod data base. While reproducing existing (Eurostat) demographic projections in aggregate outcomes, MicroWELT accounts for fertility and mortality differentials and differences in partnership careers by education. The resulting family-demographic and education projections are integrated with a longitudinal accounting framework based on the National Transfer Account (NTA) approach. National Transfer Accounts break down National Accounts by age and capture transfers within and between families and through the tax-benefit and social insurance system.

NTAs are heavily used in studies on the economic and policy implications of demographic change. In this paper, by means of a case study for Spain, we demonstrate how microsimulation can add realism to existing projections by accounting for education change as well as differential longevity and fertility patterns by education. Our analysis first reproduces a set of indicators and projection approaches found in existing literature based on aggregated NTA data. We then dis-aggregate these NTA data by education, school enrolment, and family characteristics. Combined with the corresponding detailed socio-demographic projections of MicroWELT, we explore how findings change.

KEYWORDS: MICROSIMULATION, WELFARE STATE, EDUCATION, DEMOGRAPHIC CHANGE, NATIONAL TRANSFER ACCOUNTS

JEL classification: E01, J11, P51, O57, C53

1 INTRODUCTION

MicroWELT is a dynamic microsimulation model developed alongside an international research program studying the distributional effects of four welfare state regimes - Liberal, Universalistic, Conservative, and Mediterranean - in the context of demographic change. While the project focuses on four country examples representing the four welfare state regimes -- namely Austria, Finland, the UK, and Spain – the model is designed as a modular simulation platform portable to other countries and extendable for a wide range of applications.

MicroWELT produces detailed population projections going beyond age and sex, specifically addressing education, and the effect of education on socio-demographic behaviors: differential longevity, partnership histories and fertility patterns. The model can reproduce existing population projections in aggregate outcomes but creates realistic life courses adding detail and longitudinal consistency. The model also supports a wide range of what-if analyses and hypothetical scenarios for sensitivity analysis.

MicroWELT integrates cross-sectional and longitudinal accounting (including generational accounts) based on the National Transfer Accounts (NTA) approach. NTAs depict the age profiles of income, public and family transfers, saving and consumption patterns consistent with National Account aggregates reflecting the size and aggregate pattern of welfare state transfers, together with the corresponding private reallocation (Lee and Mason, 2011, Abio et al., 2017). In the Weltransim project we further disaggregate NTA variables by education and family type, allowing for composition effects beyond age. While NTAs are a cross-sectional approach to capture the age dimension, a pseudo-longitudinal interpretation naturally arises in literature. In microsimulation terminology, it is equivalent to a static ageing approach: a re-weighting of the current population by a projected age pattern. In this respect, we complement literature by adding more dimensions, like education change, or changes in childlessness. To balance budgets over time, mechanisms must be defined. Literature has developed a range of approaches which typically ignore differences between welfare state regimes in adapting to demographic change. MicroWELT goes beyond this by a more realistic implementation of the key features of welfare state types.

MicroWELT uses the EUROMOD data as starting population and can thus directly correspond with the EUROMOD database. EUROMOD (Sutherland and Figari, 2013) is used for complementing MicroWELT for detailed distributional analysis within the studied population groups (by age, education and family) today, and to study the sustainability of the current system at future points in time; for this purpose, MicroWELT can project weights. We use EUROMOD also to identify patterns of policy changes and mechanisms to balance budgets typical for the various welfare state regimes.

In this paper we present the MicroWELT model and demonstrate how microsimulation techniques can contribute to research on intergenerational distribution based on NTA data by a case study for Spain. Our analysis first reproduces a set of indicators and projection approaches found in existing literature based on aggregated NTA data. We then dis-aggregate these NTA data by education, school enrolment, and family characteristics. Combined with the corresponding detailed socio-demographic projections of MicroWELT, we explore how findings change when adding detail to projections. For this demonstration, we have selected concepts developed in Lee & Mason (2017) and Lee et.al. (2017).

Our analysis illustrates, how disaggregation of NTAs by education and family type impacts projection results on sustainability. We interpret our findings also as rational for the use of microsimulation for NTA based research and – with the MicroWELT model – aim at providing an accessible and versatile tool for projections based on the NTA approach.

2 THE MICROWELT MODEL

MicroWELT is a socio-demographic projection model aiming at capturing some key welfare-state related patterns in education and family careers. While being able to reproduce available aggregate demographic and education projections (e.g. Eurostat, Ageing Report), microWELT adds realism in individual life courses and the differences in life-courses by education. Specifically, the model addresses life expectancy differentials, the differences in childlessness and timing of first birth, as well as differences in partnership formation and stability by education. Education is modeled combining macro projections (highest level and enrollment rates) with observed intergenerational transmission processes and individual study patterns. The model integrates NTAs and accounting routines. Keeping track of partnership status, children in the household and education, the model allows the use of NTA data broken down by family types (partnership status, presence and age of children, childlessness) and education (of parents for children and students, own education otherwise). MicroWELT produces a series of NTA indices proposed in the literature. Longitudinal accounting also allows to study welfare flows between cohorts and between education groups. Specifically, the model is designed to address the redistribution between education groups in the context of life expectancy differences. MicroWELT is implemented in Modgen, a dynamic programming language developed and maintained at Statistics Canada. It is a continuous time interacting population model supporting the (optional) alignment of a set of key aggregated demographic and education outcomes.

2.1 Data requirements

The starting population is generated from Euromod input data and various parameters are estimated directly from this data set. Another micro-data source used in the project are Labor Force Surveys. Demographic scenarios and corresponding mortality, fertility, and migration parameters

come from Eurostat population projections complemented by own estimations and projections found in literature. A full documentation of data sources together with analysis scripts will be made available on the project website. The model is designed to be portable to other countries and to a large extent parameter generation is automated.

The model allows the parallel parameterization of NTA variables by four levels of disaggregation: by age, by age and sex, by age, sex and education, and by age, sex, education and family type. Data further disaggregated by education - and by family type – are calculated alongside the Weltransim Project for Spain, Austria, the UK, and Finland. For a growing set of countries, NTA data by education are also calculated and used alongside other research project (e.g. Renteria et al., 2016 and Hammer 2015).

2.2 Fertility

MicroWELT uses age specific fertility rates from Eurostat population projections. While reproducing these macro projections (which account for differences in total fertility), MicroWELT aims at obtaining a realistic distribution of family sizes by education. This is done by a fertility module which models individual birth risks by individual level characteristics like education and parity. Empirically, the distribution of family sizes in most European countries followed a trajectory from high concentration at the beginning of the past century, to a very equal distribution in the baby boom, back to an increasing concentration, especially higher educated women having increasing rates of childlessness (e.g. Spielauer 2005, Shkolnikov et.al. 2007). These patterns vary between welfare state regimes, e.g. fewer women having a larger share of the children is typically associated with conservative regimes. We simulate birth events by using age specific fertility rates to produce the desired number of births but assign the events to the woman with the shortest random waiting time. Currently we concentrate on first birth patterns and levels of childlessness by education. In the case of Spain, the model uses first birth rates by education (own calculations) calibrated to reproduce cohort childlessness by education as presented in Reher, & Requena (2019). In the base scenario, future childlessness is kept at the current level (as estimated for the 1970 birth cohort with a spread between 17% and 27% by education). After first birth rates are met, the remaining births are distributed to mothers by age specific rates.

2.3 Mortality

The ability to quantify the impact of differential longevity by education on redistributive processes is one of the aims of the MicroWELT model. In the presented simulations we assume that the relative mortality risks by education stay constant over time, while total mortality outcomes are aligned to reproduce Eurostat mortality projections. Model parameters are period life tables by sex and estimates of the current remaining life expectancy by education at age 25 and age 65 as published by OECD ([Murtin et al. 2017](#)). Relative mortality risks are calculated at the start of the

simulation and kept over time, while the age-specific baseline risks are aligned each year to reproduce the target mortality. For estimates on the remaining life expectancy, we used Italy as proxy, as no Spanish data are available. Longevity differentials are higher for men – 3.8 years at 25 and 2.3 years at 65 between the low and high education groups – compared to 2.0 and 1.2 for women. These differentials are typical in an EU wide comparison.

2.4 Education

MicroWELT distinguishes three education levels – low, medium, and high – corresponding to compulsory education, secondary education, and post-secondary education attainments. Highest education eventually obtained is decided at birth by selecting one of two modeling approaches – or by combining them: outcome-parameters for projected age and cohort-specific distributions, and parameters for the distributions by parent’s education. When selecting the model simulating the intergenerational transmission of education, users can choose to align the aggregate outcomes to the external targets. In this case, the odds ratios between groups from the intergenerational transmission models are used to select students based on their parents’ education, while the number of graduates is calculated from the outcome parameters. Accounting for parents’ education is important when modeling NTAs by education, as the transfer a child receives (as well as education choices) depend on the education of parents. Besides the highest educational attainment, MicroWELT also models study patterns, i.e. school enrolment. Again, this is important from the NTA perspective, as we disaggregate NTAs by enrolment status.

In the case of the presented simulations, we selected the intergenerational transmission model without alignment. While Spain experienced a fast education expansion in the recent past, overall trends almost levelled off in our simulation, leading to an almost constant educational composition of future birth cohorts (0.23-0.24-0.53 for women, and 0.31-0.26-0.44 for men). The absence of an overall trend also results from the recent increase in childlessness of higher educated women, the recent education expansion not fully feeding into the education composition of mothers.

2.5 Partnerships

MicroWELT models the female partnership status according to observed partnership patterns by age, education, and the presence and age of children. Appropriate partners are matched by age, education, and childlessness. The model assumes that *cet. par.* the probability to be in a partnership does not change over time, thus all changes come from composition effects. Besides of changes due to the death of a partner, updates are performed at a yearly basis to maintaining cross-sectional consistency. Under the assumption of time-invariant patterns, the model is longitudinally consistent by education, childlessness and birth cohort – thus allows the calculation of consistent life course measures by these groups - but (currently) does not model consistent individual life-courses within these groups. The partnership status is modeled for all women within the age range

15-80, no more union formation events are modeled thereafter when it is assumed the only union dissolution is due to widowhood.

Male partners are matched by age and education. Men destined to stay childless (a model parameter by education and cohort) avoid unions with mothers unless no other men are found; if in a union at a birth of a child, they pass on their “never father” status to another (childless) men of the same age and education to meet overall childlessness rates. Concerning age differences between partners, the model tries to fit observed distributions by age. Empirically, the spread increases with age. Part of this pattern arises from re-partnering, the distribution of age differences thus differing for new partnerships compared to all observed partnerships. As the former typically cannot be observed in data, at each partnership event, the current age distribution in the simulation is compared with a target distribution and partners are picked to best close the gaps between the two distributions.

2.6 Migration

The migration modules of MicroWELT allow reproducing aggregate projections of immigration by number, age distribution, and sex – and emigration rates by age and sex. Modules for family migration and migration by a set of regions of origin are currently under development in the sister model MicroIMMI (Horvath 2019). For the illustration in this paper, migration is switched off.

2.7 Family Links

MicroWELT models nuclear families. At birth, children are linked to their mother and – if present – their father. This link to biological parents is maintained within the simulation. As the partnership status is only updated on a yearly basis, we assume that if a single mother enters a partnership within the first year of her child, the partner is also the biological partner. Until leaving home, children also keep links to their current “guardians”: in the case of a union dissolution of parents, children choose with whom to live, and if this parent enters a new partnership, a link to this new partner is established. In the case of the death of a single “guardian”, children move back to a biological parent if available, or to grandparents. Children leave home when entering a partnership, becoming parent, or at age 18, if not enrolled in school. Students stay at home up to age 25, the probability estimated from EU-SILC data.

2.8 NTA Variables, Indicators, and Accounting

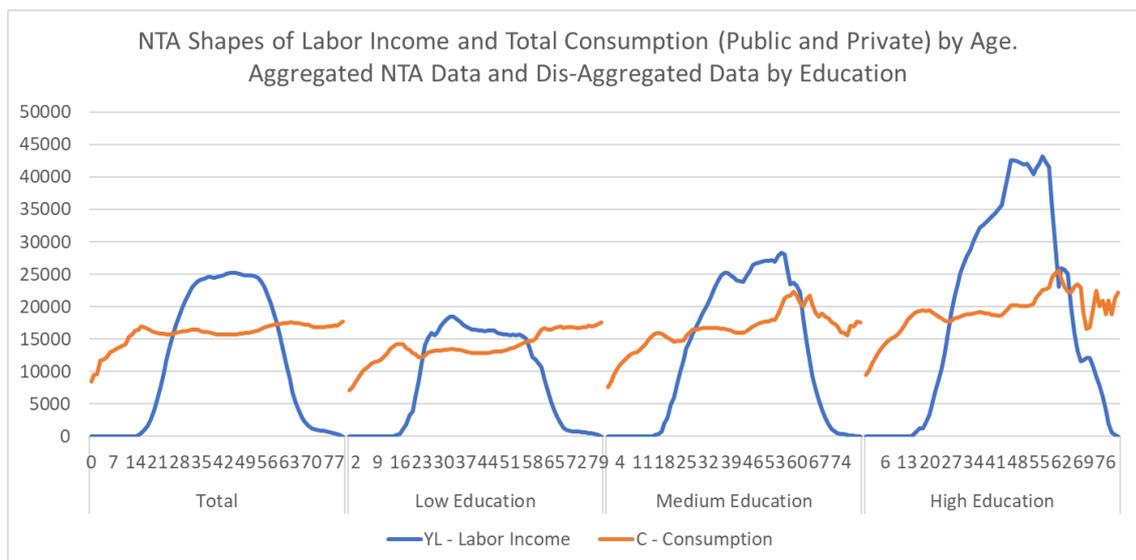
NTA data are cross-sectional age profiles breaking down national accounting variables on consumption, income, saving, and transfers. Data distinguish between private and public consumption singling out education and health. Public transfers also distinguish pension benefits. NTA data by sex are available for 50 countries, including most European countries: The AGENTA project produced NTA estimates using comparable European data. We further dis-aggregated NTA data by education and family type. MicroWELT allows the user to choose the aggregation

level, allowing to compare simulation results based on aggregated profiles with simulations accounting for composition effects along the education and family dimensions. At their most disaggregated level, NTA variables in MicroWELT are parameterized by the following population groups:

- Children age 0-16 and students age 17-25: by highest parent's education
- Non-students age 17-60: by sex, education, presence of partner and children in the household
- Persons 60+ by sex, education, partnership status, and childlessness

Figure 1 illustrates NTA shapes for labor income and consumption by education. Labor income is concentrated to work age, higher educated people reaching higher incomes, especially later into their work career and work until higher age. At their peak, the average labor income in the high education group is almost 3 times higher than in the low education group. Consumption is much smoother; also, the differences between education groups are smaller, adult people in the high education group consuming around 50% more than in the lowest group.

Figure 1: NTA age profiles of labor income and consumption



NTA data provide a detailed picture, how resources are re-distributed by means of public and private transfers and asset re-allocation. MicroWELT implements a set of 19 NTA variables (see Appendix). The NTA based indicators and accounting approaches used in the application examples of this paper are implemented in separate (and optional) modules, each corresponding to the referenced publication introducing the concepts.

3 APPLICATION EXAMPLE: NTA INDICATORS AND NET PRESENT VALUE OF PUBLIC TRANSFERS IN SPAIN

In this section we reproduce a set of indicators and projection approaches found in existing literature based on aggregated NTA. We then explore, how results change when using dis-aggregated NTA data accounting for differences by education and family type. We find, that the impact of adding some basic detail and realism to existing projections is considerable, providing a strong rationale for the use of dynamic microsimulation to inform projections based on the NTA approach. All presented scenarios reproduce Eurostat fertility and mortality projections (in the aggregate) but set international migration to zero.

The simulations start from the 2010 EUROMOD data with a sample size of 36,594 persons. For each scenario we run 10 replicates of a population of 250,000 persons each, sampled from the starting population database. While the simulation size of 2.5 Million actors eliminates most Monte Carlo variation, it cannot eliminate the starting population randomness of the underlying population sample. Dis-aggregated NTA variables rely on small samples and – for single years of age - are subject to high parameter uncertainty. As all presented indicators and accounting concepts aggregate over the population a/o the whole life-course, we expect that these errors cancel out. The cross-sectional correspondence between published aggregate profiles, the weighted sum of dis-aggregated profiles, and simulated aggregates was validated.

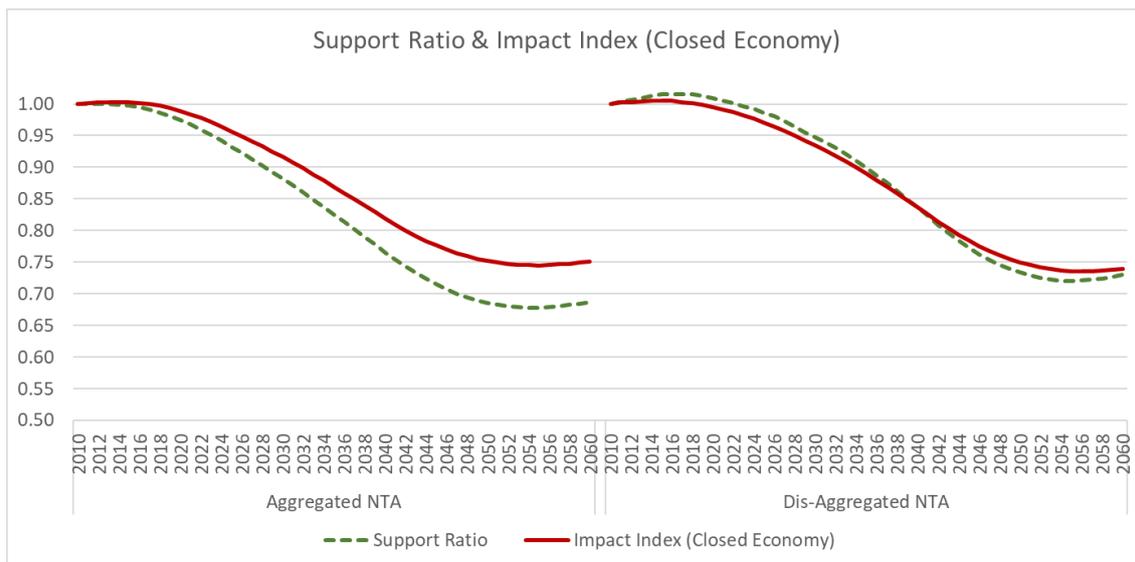
3.1 Sustainability Indicators: Support Ratio and Impact Index

One of the most widely used indicator in NTA literature is the Support Ratio, i.e. the number of effective producers per effective consumer determined by the population age distribution and the age profiles of per capita consumption and labor income as observed today (Lee & Mason 2014). It refines the simple demographic dependency ratio by accounting for the age profiles of consumption and labor. As an index set to 1 in the base year, the measure shows the change in the relationship between available labor to the current level of consumption in absence of economic growth or changes in the age profiles of consumption and labor. As depicted in Figure 3, basing projections on dis-aggregate NTAs lead to a less pronounced decline of the Support Ratio, a result mainly driven by the educational expansion, higher educated people providing more labor input.

The Impact Index proposed by Lee & Mason (2017) further accounts for changes in wages resulting from the changing labor supply in relation to capital due to demographic change. It measures the change in the relationship between consumption and the current consumption level. As in ageing societies – based on aggregated NTA data – labor becomes scarcer, wages increase, softening the consequence of ageing on future consumption. Calculation of the Impact Index requires a simple economic growth model based on a Cobb Douglas production function without economic growth. Initial capital by age is calculated indirectly from the age shape of capital income

assuming an initial interest rate (3% in our example). The resulting capital endorsement by age is assumed to stay constant, with wages and interest rates adapting to population change (assuming a closed economy). In Figure 3 we depict the Impact Index comparing simulations based on aggregated versus dis-aggregated NTA data. Due to the higher labor input accounting for the changing educational composition - and thus a lower impact of population ageing on wages - the difference between the two indices disappears when using dis-aggregated NTA data.

Figure 2: MicroWELT simulation projections of support ratios and the impact index by calendar year.

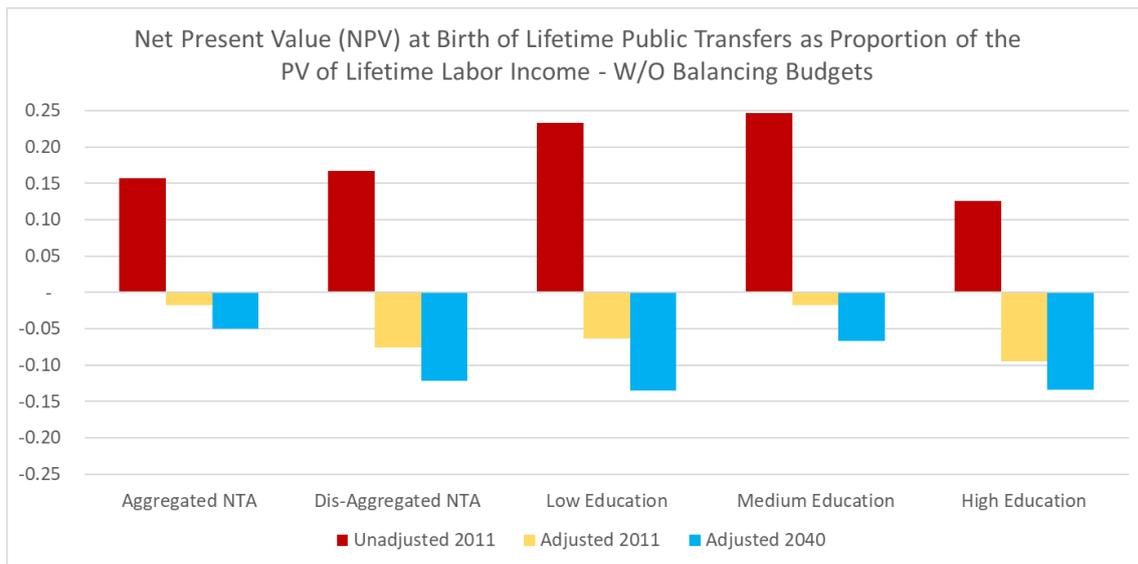


3.2 Net Present Value of Public Transfers

Following the approach developed in Lee et. al. (2017) for studying the intergenerational dimension of transfers further requires an assumption on economic growth and on an appropriate discount factor. Like in the original study, we use a discount factor of 3% and set the economic growth to 1.5% per year. Before adjustments for balancing budget, all incomes and transfers are assumed to grow at the same rate.

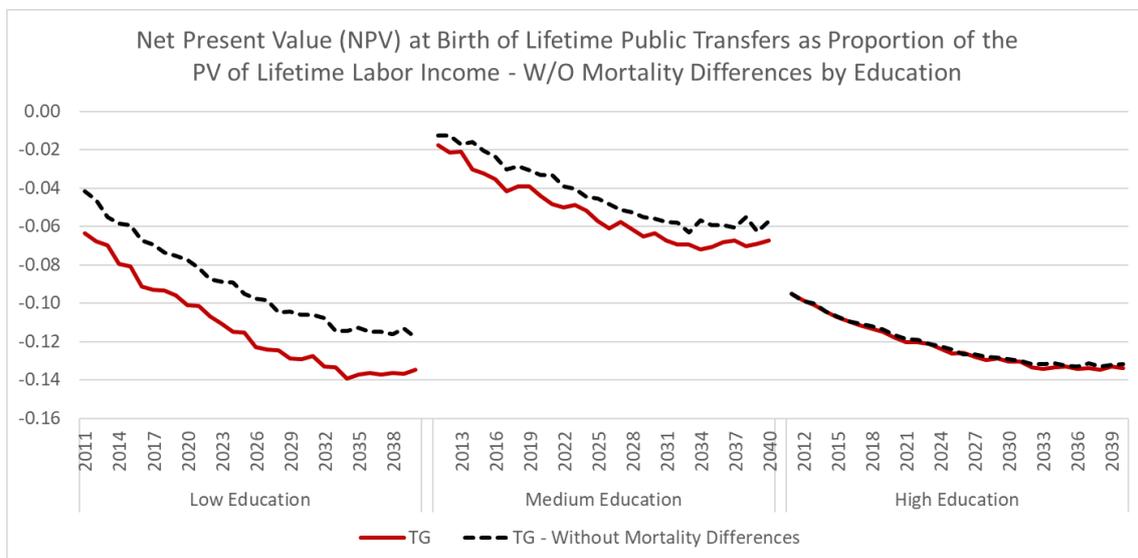
Figure 4 depicts the net present value (NPV) at birth of public transfers. Without adjustments of taxes and benefits to balance budgets, the NPV of public transfers amounts to 15% of the present value (PV) of the labor income of a cohort. For a 2011 birth cohort, disaggregation of NTAs has only a small effect on this measure, but calculating the NPV by education group, we see that the NPV expressed as percentage of the PV of labor income is about double as high for the lower two education groups compared to the high education group. As in Lee et. al. (2017) we assume a symmetric adjustment of transfers for balancing budgets each year; i.e., taxes are increased by the same extent as benefits are decreased. After this yearly adjustment, the NPV turns negative. Using aggregated NTA data, the NPV for a 2011 birth cohort is still close to 0 for the 2011 birth cohort but further decreases for a 2040 birth cohort. Using dis-aggregated NTA data, the effect is more dramatic.

Figure 3: Net Present Value of public transfers



Patterns vary by education, the low education group being most hit by the adjustments. In the presence of mortality differentials by education, we can expect an increase of average pensions as those with the higher pensions on average live longer. Consequently, a higher adjustment of public transfers is required to balance budgets, which negatively affects all education groups.

Figure 4: The effect of mortality differences by education on the NPV of public transfers



For the lower educated group, shorter life and the required additional adjustments have a negative impact on the NPV of public transfers. For the high education group, longer lives and additional adjustments have opposite effects. Running a scenario without mortality differences shows that effects cancel themselves out in the high education group, while the low educated group loses more than 2% of the PV of lifetime labor income.

4 SUMMARY AND OUTLOOK

The presented first results for Spain only cover a part of the aspired strengths of MicroWELT which can be summarized by four key dimensions:

First, MicroWELT provides a tool for adding detail to existing NTA based projections for the study of redistributive patterns between education groups, accounting for longevity and fertility differentials by education. Detail refer to both the aggregation level of NTA data, but also the support of realistic country-specific mechanisms of balancing budgets. While we limited the presented analysis to public transfers, the NTA approach – and MicroWELT - allow for analysis of intra- and inter household transfers giving a more complete picture of redistributive mechanisms. A second related concept complementing the NTA approach are National Time Transfer Accounts (NTTA) which will be incorporated into the model.

Second, MicroWELT provides a tool supporting comparative studies. While the project focuses on four country examples representing four welfare state regimes (Austria, Finland, the UK, and Spain) the model is easily portable to many European countries.

Third, MicroWELT provides a modeling platform extendable for a wide range of applications going beyond the NTA approach, e.g. by explicit modeling of policies and behaviors, like labor market participation and earnings, enabling distributional analysis also within the NTA population groups.

Finally, MicroWELT aims at complementing (and communicating with) EUROMOD.

ACKNOWLEDGEMENTS

The model is part of the (2017-2020) WELTRANSIM project, funded by the Joint Programming Initiative “More Years, Better Lives” and coordinated by the University of Barcelona. MicroWELT is developed at the Austrian Institute of Economic Research; other project partners are the, the Finnish Institute for Economic Research, and the Finnish Center for Pensions.

The model shares some code with DYNAMIS-POP (Spielauer & Dupriez 2019) – an open-source portable microsimulation modeling platform for developing countries, and with DypenSi, a Slovenian pension microsimulation model (Kump et.al 2017). Like DYNAMIS-POP we envisage MicroWELT as a modular microsimulation platform (for EU countries); branches of the model are currently used as starting point of a model for employment projections in Austria as well as for a study of the economic integration of immigrants in Austria (Horvath 2019).

AVAILABILITY

The model code as well as analysis scripts for parameter generation and estimation are available on the MicroWELT project website www.microwelt.eu. European NTA data by age and sex are available at the AGENTA project site agenta-project.eu. The dis-aggregated data developed as part of the WELTRANSIM project will be made available at the WELTRANSIM project website weltransim.eu.

REFERENCES

- Abío, Gemma, Concepció Patxot, Miguel Sánchez-Romero, Guadalupe Souto (2017) *The welfare state and demographic dividends*. Demographic Research. Volume 36, Article 48. <http://www.demographic-research.org/Volumes/Vol36/48/>
- Hammer, B. (2015) National Transfer Accounts by Education: Austria 2010, AGENTA Working Paper 2/2015.
- Horvath, T. (2019) Long-term Integration of Immigrants in Austria. A Dynamic Microsimulation Model. Austrian Institute of Economic Research. Project description at: https://www.wifo.ac.at/forschung/projekte/aktuelle_projekte?detail-view=yes&publikation_id=59043
- Kump, N, B Majcen, J Sambt, A Lotric Dolinar, M Spielauer, M Verbic, R Spruk (2017) *Dinamichni Mikrosimulacijski Pekojninski Model*. Monograph. EkonomIERa, ISSN 2630-2896. Socialni razvoj. ISBN 978-961-6906-43-2.
- Lee, Ronald & Andrew Mason (2017) Some Economic Impacts of Changing Population Age Distributions - Capital, Labor and Transfers - Agenta Keynote, WP.
- Lee, Ronald, David McCarthy, James Sefton, Jože Sambt. (2017) *Full Generational Accounts: What Do We Give to the Next Generation?* - Population & Development Review.
- Lee, Ronald and Andy Mason (2011) Population Aging and the Generational Economy: A Global Perspective. Edward Elgar Publishing, Northampton, Massachusetts.
- Murtin, F. et al. (2017), "Inequalities in longevity by education in OECD countries: Insights from new OECD estimates", OECD Statistics Working Papers, 2017/02, OECD Publishing, Paris.
- Rentería E., Mejía-Guevara I, Patxot C, Souto G. (2016). "The effect of education on the demographic dividend", Population and Development Review, 42, 4, 651-671.
- Spielauer, Martin (2005) Concentration of Reproduction in Austria: General Trends and Differentials by Educational Attainment and Urban-Rural Setting. Vienna Yearbook of Population Research Vol. 3 (2005), pp. 171-195
- Spielauer, Martin, Olivier Dupriez (2019) *The DYNAMIS-POP dynamic microsimulation model*. Project website: dynamis.ihsn.org

- Shkolnikov, V.M.; Andreev, E.M.; Houle, R.; Vaupel, J.W. (2007) *The concentration of reproduction in cohorts of women in Europe and the United States*. Population and Development Review 33(1): 67-100
- Sutherland, H., Figari, F., *EUROMOD: The European Union Tax-benefit Microsimulation Model*. International Journal of Microsimulation, Vol. 6, No. 1, 2013, pp. 4-26
- Reher, David & Miguel Requena (2019) Childlessness in Twentieth-Century Spain: A Cohort Analysis for Women Born 1920–1969, European Journal of Population (2019) 35:133–160
<https://doi.org/10.1007/s10680-018-9471-7>

APPENDIX

A.1. NTA Variables in MicroWELT:

- Private Consumption Education (CFE)
- Private Consumption Health (CFH)
- Private Consumption other than Education and Health (CFX)
- Public Consumption Education (CGE)
- Public Consumption Health (CGH)
- Public Consumption other than Education and Health (CGX)
- Public Transfers Pensions, Inflows (TGSOAI)
- Public Transfers Other Cash Inflows (TGXCI)
- Public Transfers Other In-Kind Inflows (TGXII)
- Public Transfers Education Inflows (TGEI)
- Public Transfers Health Inflows (TGHI)
- Public Transfers Outflows (TGO)
- Net Interhousehold Transfers (TFB)
- Net Intrahousehold Transfers (TFW)
- Private Saving (SF)
- Public Saving (SG)
- Labor Income (LY)
- Private Asset Income (YAF)
- Public Asset Income (YAG)

A.2. Calculation of the Support Ratio and the Impact Index for a closed economy:

Variables:

- $y_l(x)$ Average labor income age x
- $y_k(x)$ Average capital income age x
- $P(x)$ Population age x
- $i(x)$ Average savings age x
- $s(x)$ Saving rate age x - constant
- $c(x)$ Average Consumption age x (reference values for calculation of N) - reference
- L Labor
- K Capital
- I Saving
- S Saving rate
- C Consumption
- r Interest rate
- w Wage
- Yl Labor Income
- YK Capital Income
- Y Total Income Yl+Yk
- α Alpha – constant
- N Effective Consumers (population-weighted base-year consumption)
- $l(x)$ Average labor age x – constant
- $k(x)$ Average capital age x – constant

Cobb Douglas production function:

- $Y = L^\alpha K^{(1-\alpha)}$
- $Y = wL + rK$
- $w = \alpha Y / L$
- $r = (1-\alpha) Y / K$
- $Yl = wL = \alpha Y$
- $Yk = rK = (1-\alpha)Y$

Known:

- $y_l(x)$ NTA data of reference year
- $y_k(x)$ NTA data of reference year
- $i(x)$ NTA data of reference year
- $c(x)$ NTA data of reference year
- $P(x)$ NTA data of reference year

- r parameter for reference year (used also to estimate stock from capital income flow)

Calculated for initial year:

- $Y_l = \sum y_l(x) * P(x)$
- $Y_k = \sum y_k(x) * P(x)$
- $Y = Y_l + Y_k$
- $\alpha = Y_l / Y$
- $K = Y_k / r$
- $L = (Y / K^{(1-\alpha)})^{(1/\alpha)}$
- $w = Y_l / L$
- $s(x) = i(x) / (y_l(x) + y_k(x))$
- $l(x) = y_l(x) / w$
- $k(x) = y_k(x) / r$
- $N = \sum c_l(x) * P(x)$

Simulation: calculate for an updated population by age $P(x)$

- $L = \sum l(x) * P(x)$
- $K = \sum k(x) * P(x)$
- $Y = L^\alpha K^{(1-\alpha)}$
- $Y_l = \alpha Y$
- $Y_k = (1-\alpha)Y$
- $w = \alpha Y / L$
- $r = (1-\alpha) Y / K$
- $N = \sum c(x) * P(x)$
- $y_l(x) = w * l(x)$
- $y_k(x) = r * k(x)$
- $C = \sum (1-s(x)) * (y_l(x) + y_k(x)) * P(x)$

Indices

- $SR = L / N$ Support Ratio
- $IMP = C / N$ Impact Index